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Abstract

This paper presents a mathematical analysis of a series of geometrical abstract artworks by Portuguese author Almada Negreiros (1893-1970), understood in the context of the author's search for a *canon*.

After a brief description of Almada's work in the frame of 20th century visual arts, we examine the mathematical elements in three of his works: illustrations for a newspaper interview, two drawings from a collection called *Language of the Square* and his last visual work, the mural *Começar*.

The analysis revealed that some of the author's geometrical constructions were mathematically exact whereas others were approximations. We used computer based drawings, along with mathematical deductions to examine the constructions presented in the aforementioned works, which we believe to be representative of Almada's geometric statements. Our findings show that, albeit limited by the self-taught nature of his endeavor, the mathematical content of these artworks is surprisingly rich.

The paper is meant to be an introduction to Almada's work from a mathematical point of view, showing the importance of a comprehensive study of the mathematical elements in the author's body of work.

1 Introduction

José de Almada Negreiros (São Tomé and Príncipe, 1893 – Lisbon, 1970) was a key figure of 20th century Portuguese culture, in both visual arts and literature. See Figure 1 for a photo of Almada.¹

As a member of the group *Orpheu* (composed by the new literary generation of the beginning of the century) and by proclaiming himself a futurist, he clearly demonstrates from a young age his desire to break with the academic traditions of his time. In the beginning of his artistic and literary career he made himself noticed by his active and openly critical stance on the Portuguese art scene of the time, with drawings, exhibitions, collaborating in literary journals, or his famous Anti-Dantas Manifest (1915). In this text he reacts to reactionary positions in the Portuguese art scene. Even if his visual artwork is not easily associated with futurism, being closer to cubism or geometric abstractionism, Almada remains a crucial figure of Portuguese modernism.

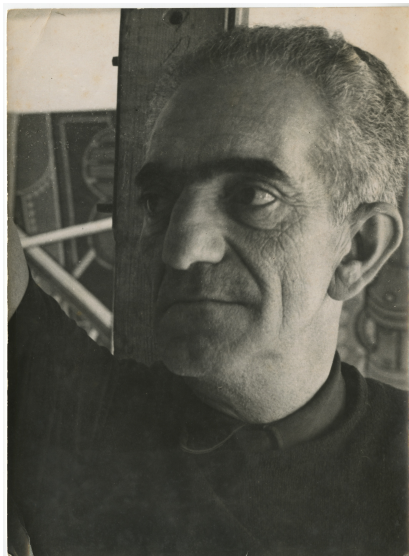


Figure 1: Almada Negreiros in the late 1940s

In his early years, Almada was close to Amadeo de Souza-Cardoso² and Santa-Rita Pintor³, which were the main figures in Portuguese modern art. Although Amadeo and Santa-Rita died in 1918, Almada continued to work intensely for many decades. Over the years, his initial restless attitude gave way to a more lyrical approach in his works, both in visual art and literature.

¹The artist himself chose to use just the name “Almada”. His signature can be seen in some of the artworks presented in this paper.

²Amadeo de Souza-Cardoso (1887-1918): Probably the most internationally recognized Portuguese modernist painter, he left an extensive body of work that articulates Cubism, Futurism and Expressionism.

³Santa-Rita Pintor (1889-1918): A member of the first generation of Portuguese modernist painters, he died prematurely leaving orders to have his work destroyed. Few examples of his artwork remain.

After a short stay in Paris (1919-1920) he resided for some time in Madrid (1927-1932) where he began articulating his work with architecture, something he would continue to do for many years with different media. Returning to Portugal, where he would spend the rest of his life, he marries the painter Sarah Afonso⁴ and carries out a long list of public works in collaboration with architects, namely Pardal Monteiro⁵. Among these the most remarkable are the murals of the buildings at the maritime stations of Rocha Conde de Óbidos and Alcântara, in Lisbon. Overall, Almada leaves a vast legacy, which includes poetry, plays, lectures, painting and public art such as murals, frescoes, stained glass windows and tapestries.

Over the years, his work came to incorporate more geometric abstraction—from a very early stage geometry was one of the author’s admitted passions (notably in the understanding and analysis of Portuguese renaissance painting, as we will see in the next section). In the 1950s his artwork begins to have a strong geometric tone bearing some relations, in form and artistic intention, to Mondrian ou Le Corbusier. In the 1960s he publishes several interviews (cf. [Va60]) on the importance of geometry in art, proposing a universalistic theory on the matter, which he called a canon. Motivated by the search for this canon, to be found in various artistic manifestations throughout time, Almada devoted himself, for decades, to research on geometry. His last piece, *Começar*, which will be studied in this paper, is a remarkable conclusion of this search.

Almada Negreiros’ estate is currently undergoing inventory and analysis by a multidisciplinary team that includes researchers from several areas, such as literature, fine arts, history and mathematics. This research made possible the analysis of artworks that were, up to now, unknown to the public.

To our knowledge, only the books [Fr77] and [Ne82], the article [Co94], and the thesis [Va13] presented some mathematical analysis of Almada’s work, some of the material about *Começar* is already studied in some of these works, but not with the detail we present here. We have published an analysis of some drawings in [CF14], which is a bilingual publication (Portuguese and English). The material about the *Language of the Square*

⁴Sarah Afonso (1899-1983): From the second generation of Portuguese modernist painters, the wife of Almada Negreiros studied painting in Paris during the 1920s but decided to stop pursuing her career in 1940.

⁵Pardal Monteiro (1897-1957): One of the most important Portuguese architects of the first half of the twentieth century. He was responsible, along with a small group of other architects, for the implementation of modernist architecture in Portugal.

we present is original.

Other than these, we know of no other published studies of the mathematical elements in Almada's visual work.

2 Almada and Renaissance Art

Almada's interest in geometry stemmed from his studies of Portuguese renaissance visual art. He was particularly interested in two masterpieces: a Portuguese renaissance polyptych (composed of six panels), the *Painéis de S. Vicente*, by Nuno Gonçalves, and *Ecce Homo*, originally also attributed to Nuno Gonçalves, now known to be a later work. Both of these are still on display at the Portuguese *Museu de Arte Antiga*.

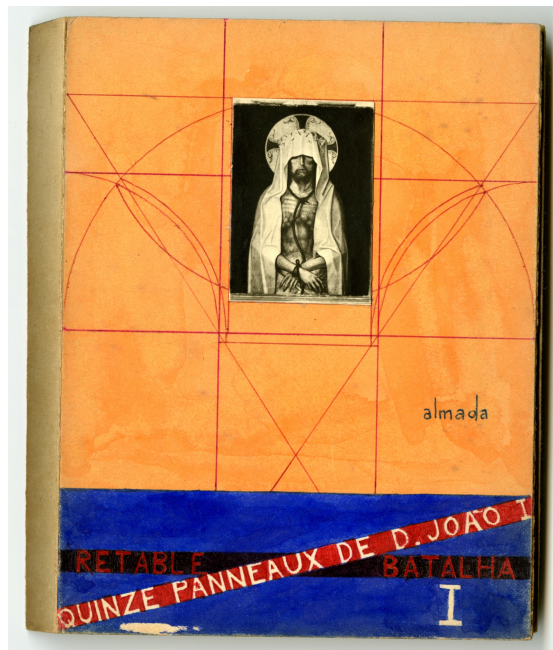


Figure 2: Cover of a notebook with a photograph of *Ecce Homo*

Figure 2 is the cover of one of Almada's notebooks, with a photo of the *Ecce Homo* and some geometric studies.

This fascination with *Ecce Homo* and the *Painéis de S. Vicente* led Almada to a compositional analysis of these pieces, using circles, squares, rectangles, and other geometrical constructions, superimposed on the paintings (as can be hinted from the geometric lines on the notebook cover).

Some of these studies on perspective actually led to a redefinition of the way the polyptych was presented: nowadays, the six panels are presented as an ensemble, instead of being divided in two triptychs.

This analysis was deeply rooted in geometric considerations. It started probably in the 1920s and lasted until the end of his life, in 1970. As his investigations developed, Almada postulated the existence of a *canon*, present in all art throughout history, manifesting itself in specific rules in each artistic period. He used several expressions to refer to this canon, the most famous one being “relation nine/ten”.

It is not clear what were the contents of this canon, one can only guess what they were from the art works produced and some brief (and often obscure) explanations by Almada. For instance, here’s how he described the elements of the canon in an interview, [Va60]:

The simultaneous division of the circle in equal and proportional parts is the simultaneous origin of the constants of the relation nine/ten, degree, mean and extreme ratio and casting out nines.

As Almada became more interested in this canon, he

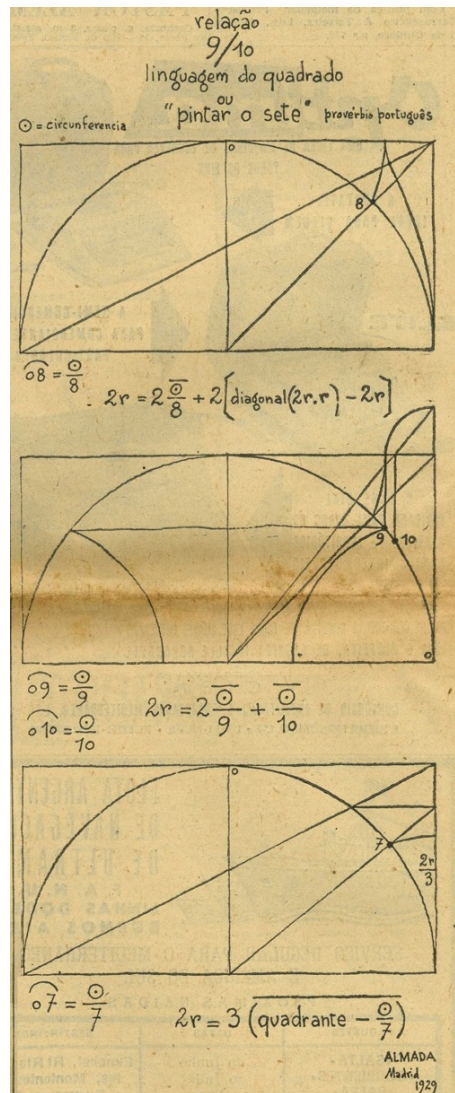


Figure 3: Drawings from 1929, published in the newspaper *Diário de Notícias* in 1960 illustrating the interviews [Va60]

started to produce more geometrically based artworks, some of them meant to reveal this canon, directly and without explanation. As a first example, Figure 3 presents three drawings of canonic elements. The drawings are from 1929 and are included in the interviews [Va60].

The title reads

relation
9/10
language of the square
or
“paint the seven”⁶

In these drawings Almada presents three relations between some measurements in the semicircle and the circumscribed rectangle.

The first one states that the chord of the 8th part of the circle, added to the difference between the diagonal and the side of the rectangle is equal to the radius of the circle.

The second one presents constructions for the 9th and 10th parts of the circle, stating that the diameter is equal to the chord of the 10th part plus two times the chord of the 9th part.

The third one starts by constructing the 7th part of the circle, stating afterwards that the chord of difference between the quarter circle and the 7th part of the circle is two-thirds of the radius.

As we will see, all these three statements are approximations. Even without any computations, one can see that the two last ones can only be approximations because of the following theorem.

Gauss-Wantzel’s Theorem. *The division of the circle in n equal parts with straightedge and compass is possible if and only if*

$$n = 2^k p_1 \dots p_t$$

where p_1, \dots, p_t are distinct Fermat primes.

A *Fermat prime* is a prime of the form $2^{2^m} + 1$. Presently, the only known Fermat primes are 3, 5, 17, 257 and 65537.

This result implies that the 9th and the 7th parts of the circle cannot be determined with straightedge and compass, since 7 is not a Fermat prime

⁶This is a Portuguese expression, with a meaning similar to “Paint the town”. Almada’s own interpretation was “to produce wonders”.

and $9 = 3 \times 3$, the Fermat prime 3 appears twice in the factorization of 9. Therefore, the last two constructions cannot be exact, as they include the chords of the 7th and 9th parts of the circle.

As for the first construction, one can also conclude that it is approximate, using the following formula for the chord of the n -th part of the circle of radius r :

$$\text{chord}(n) = 2r \sin(\pi/n). \quad (1)$$

The value of $\sin(\pi/8)$ can be found using elementary complex analysis, and it equals $\frac{1}{2}\sqrt{2 - \sqrt{2}}$, so $\text{chord}(8) = \sqrt{2 - \sqrt{2}}r$. The difference between the diagonal and the long side of the rectangle is $\sqrt{5}r - 2r = (\sqrt{5} - 2)r$, so

$$(\text{chord}(8) + \sqrt{5} - 2)r = (\sqrt{2 - \sqrt{2}} + \sqrt{5} - 2)r = 1.001434r.$$

The approximations in the other constructions are of the same order of magnitude and can be easily calculated with any dynamic geometry application, such as geogebra.

So, in the present case, all three constructions are approximations, but in the next section we will present some exact ones. It is not clear whether Almada was aware of this, as he presents them all without explanations, and without distinguishing the exact ones from the approximate ones. There is another quote from Almada, also in [Va60], which may refer to this: “Perfection contains and corrects exactness”.

The presence of mathematics in Almada’s work is thus justified by the nature of the author’s program: to find and reveal the canon underlying all art. One that, in his own words “is not the work of Man, but his possible capturing of immanence. It is the initial advent of epistemological light” (interviews [Va60]). Mathematics is particularly appropriate for this project, as its constructions are abstract and general, not directly connected to any particular art period.

In this work we do not wish do dwell on the philosophical and artistic issues that would develop from Almada’s postulate of the existence of this canon. We are primarily interested in analysing the geometric artworks from a mathematical viewpoint (as we have just done).

3 Language of the Square

Almada expanded this type of simple geometric drawings in a collection, dating probably from the 1960s, which he called *Language of the Square*.

$EO/DE = 1/\phi$. Since $\phi^2 - \phi - 1 = 0$, we have

$$\frac{1}{\phi} = \phi - 1 = \frac{1 + \sqrt{5}}{2} - 1 = \frac{\sqrt{5} - 1}{2},$$

so we need $EO/DE = (\sqrt{5} - 1)/2$.

Since the triangles DEO and ACO are similar, we have

$$\frac{EO}{DE} = \frac{OC}{AC} = \frac{OB - BC}{AC} = \frac{OB}{AC} - \frac{BC}{AC} = \frac{AB}{AC} - \frac{BC}{AC}.$$

Now we use the similarity of triangles ACB and FGB , noting that in both of them the larger leg measures twice the smaller, since the rectangle $BPFG$ is a half square. This means that $BC = (1/2)AC$ and

$$AC^2 + \left(\frac{1}{2}AC\right)^2 = AB^2 \Leftrightarrow \frac{AB^2}{AC^2} = \frac{5}{4}.$$

Therefore

$$\frac{EO}{DE} = \frac{AB}{AC} - \frac{BC}{AC} = \frac{\sqrt{5}}{2} - \frac{1}{2} = \frac{1}{\phi}.$$

For the 5th and 10th parts of the circle, we will use the formula for the chord, which we have already presented. We need the sines of $\pi/5$ and $\pi/10$, which can be determined exactly:

$$\sin(\pi/5) = \frac{\sqrt{2(5 - \sqrt{5})}}{4} \quad \sin(\pi/10) = \frac{\sqrt{5} - 1}{4} = \frac{1}{2\phi}.$$

According to formula (1), page 7,

$$\text{chord}(5) = \frac{\sqrt{2(5 - \sqrt{5})}}{2}r \quad \text{chord}(10) = \frac{r}{\phi}.$$

Therefore, all that remains to be proved is that

$$OD = \frac{\sqrt{2(5 - \sqrt{5})}}{2}r \quad \text{and} \quad OE = \frac{r}{\phi}.$$

Both assertions are easy consequences of the fact that $DEOQ$ is a golden rectangle.

Figure 6 sketches the standard constructions for the 5th part of the circle and the golden rectangle.

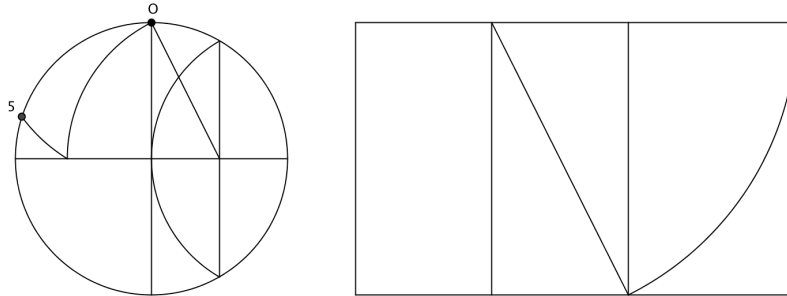


Figure 6: Standard constructions for the 5th part of the circle and the golden rectangle

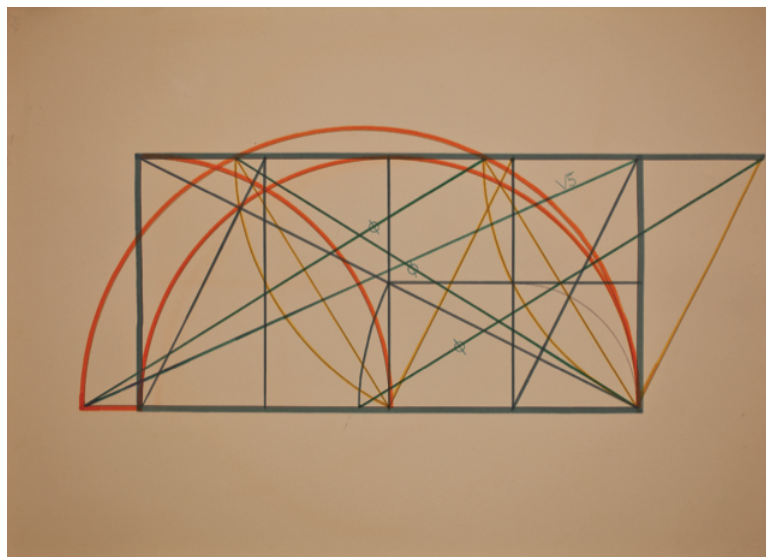


Figure 7: Another drawing from the collection *Language of the Square*

To finish this section, we present, in Figure 7, another drawing from the same collection in which the previous construction appears.

All lines marked ϕ are diagonals of golden rectangles, and the line marked $\sqrt{5}$ is the diagonal of a rectangle in which the length of the larger side is $\sqrt{5}$ times that of the smaller one. Also, one can see an intersection of four lines, two straight lines and two arcs of circle, in the top left (we invite the reader to check that the four lines do intersect). The two blue straight lines in this

intersection are called *reciprocal diagonals* in Hambidge's book [Ha67]. These are perpendicular lines that can be used to construct similar rectangles, which have these lines as diagonals. Hambidge also makes an extensive study of rectangles of given proportions (ϕ and $\sqrt{5}$ are among them) applying these to the analysis of art (especially of greek vases). Almada acknowledged Hambidge's influence in his own work, albeit claiming originality in his own constructions.

4 Começar

Finished in 1969, a year before Almada's death, *Começar (To begin)* is a colored sunk relief, on marble, found on Calouste Gulbenkian Foundation's main atrium wall, in Lisbon. It assembles, anthologically, on a composition of impressive size (about 2 metres by 13 metres), many of the author's discoveries on this matter, see Figure 8.



Figure 8: *Começar*, mural, 1968/69

Some of the structures and constructions presented in *Language of the square* appear here, woven into a very intricate network of lines. Some of this geometric material appeared in a tapestry from 1958, simply called *Número*

(*Number*). Figure 9 presents a detail of this tapestry, showing some of the geometric elements.

In this fragment of the tapestry we see some of what Almada called “epochal manifestations of the 9/10 relation”, elements of art that follow rules inspired by the canon. From top to bottom, we see several elements accompanied by geometric descriptions. The two top ones are a Babylonian vase and an element from a frieze in a greek palace in Knossos. Afterwards we find the Pythagorean *Tetracys* (an arrangement of ten sticks in a triangle) and a right triangle with sides measuring 3, 4 and 5 — Almada calls it the 3.4.5 triangle.

Right below we find a geometric construction of a square and a triangle inscribed in a circle. This is Almada’s interpretation of the so-called *Point of the Bauhütte*. It refers to a four verse stanza (which Almada found in a text by architect Ernest Mössel), going back to a medieval guild of masons called the Bauhütte. The stanza refers to a point, critical for the work of stone masons, which was kept a secret, and is stated as follows in [Gh77, p. 120]:

A point in the Circle
 And which sits in the Square and in the Triangle.
 Do you know the point? then all is right,
 Don’t you know it? Then all is vain.

The stanza does not give a clear construction of the point (probably on purpose). Usually it is understood as referring to a point common to a circle, a square and a triangle, the two latter ones inscribed in the circle. This is the case with Almada’s construction, which we will analyse below in more detail.

Finally, we have a 16-pointed star which Almada attributes to Leonardo da Vinci, called the *Figura superflua ex errore*. It is also represented in *Começar*, where only half the figure is visible.



Figure 9: Detail of the tapestry *Número*, 1958.



Figure 10: Detail on the left of the mural *Começar*

chord of the 10th part:

$$2r = 2\frac{\overline{\odot}}{9} + \frac{\overline{\odot}}{10}$$

In the drawing, Almada states that OM has the same length as the chord of 9th part of the circle ($M^{\wedge}9$) and that $OM = MN$, via the half circle $O^{\wedge}N$. Finally, arc $N^{\wedge}10''$ states that NO'' is the chord of the 10th part of the circle, thus illustrating his statement.

As we have seen, the 9th part of the circle cannot be exact, in this construction the error is $0.006954r$. All the remaining constructions (the 10th part of the circle and the equality of OM and MN) are exact.

On the rightmost part of the panel we find an area with a very dense

As we can see these elements are displayed in chronological order, in an attempt to demonstrate the influence of the canon in art throughout history.

We now go back to to *Começar*, focusing on the leftmost and rightmost parts of the mural.

Figure 10 details the left side, Figure 11 reproduces the drawings, preserving the original notation for some points.

Among other elements, we find a regular pentagon, drawn in black, inscribed in a circle. With this figure Almada comes back to one of his canonical relations, already presented in the drawings of 1929, which states that the diameter of the circle is twice the chord of the 9th part plus the

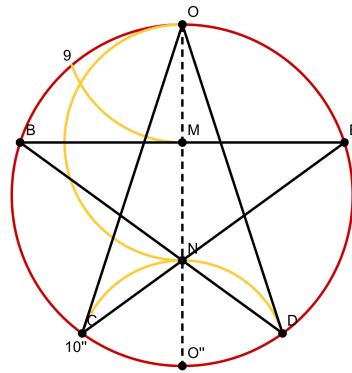


Figure 11: A reproduction of the drawing in Figure 10

mesh of lines, see Figure 12. Again, we will stick to the drawings in black, a triangle and a square, both inscribed in a circle, a representation of the Point of the Bauhütte. Figure 13 reproduces the drawing.

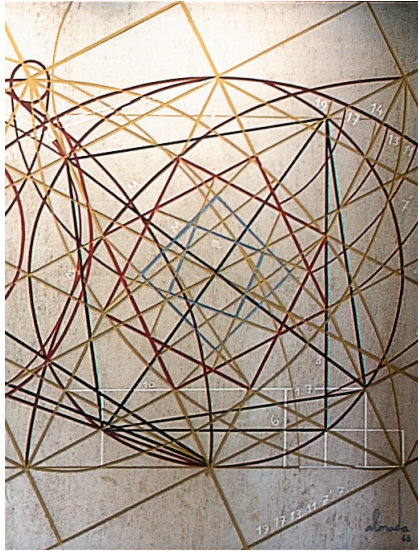


Figure 12: Detail on the right of the mural *Começar*

However, Almada himself shows us a much easier way of proving this, in one of his studies for the construction, using grid paper, see Figure 14. Notice that there is even a reference to this grid in the mural, near the bottom, which can be seen in Figure 12.

In the first of the drawings, Almada states that point E , in our previous figure, is actually a point on the grid lines. Let E' be the point on the grid lines of this first drawing, corresponding to point E .

It is easy to see that E' is the diagonal, by considering the two grid squares to its top left. On the

The diagonal of the rectangle $ABDJ$ determines point E in the circle, from which an inscribed square is drawn. From point H we now draw a vertical line HI , followed by a horizontal line IF , thus constructing a right triangle inscribed in a semicircle.

The remarkable thing is that this is a 3.4.5 triangle. This means, of course, that if we take as measurement unit a third of the shorter leg IF , then the longer leg HI will measure 4 and the hypotenuse FH will measure 5. One can prove that this is the case by computing the tangent of angle $\angle HFI$ and proving it is equal to $4/3$ (one has to compare this angle to angles $\angle JCE$, $\angle JAD$ and $\angle ADB$, knowing that $\tan(\angle ADB) = 1/2$).

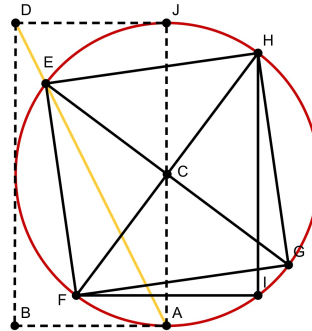


Figure 13: A reproduction of the drawing in Figure 12

other hand, if we imagine two coordinate axes through the center of the circle, with a measurement unit equal to the side of the square of the grid, then E' has coordinates $(-4, 3)$.

Since the circle has radius 5, by the Pythagorean theorem, the point lies in the circle. Therefore, it is the intersection point of the circle and the line.

With similar considerations, one would conclude that point G (in our figure) is also on the grid lines, and therefore the triangle GHE has sides that comprise 8, 4 and 10 squares of the grid. It is therefore a 3.4.5 triangle!

In the third drawing Almada has yet another reference to the number seven, stating that the four sides of the square are diagonals of grid rectangles of sides 1 and 7.

5 Conclusions

The analysis we have presented here is part of an ongoing inventory process of Almada Negreiros' work. His quest for the canon involved research primarily on geometric principles that the author considered to underlie all art and that could be immediately understood by every person, with no need for explanation. These propositions are interesting from the mathematical point of view because they establish unexpected connections between apparently disparate concepts.

As we have noticed, is not clear if Almada was aware that some of these constructions were approximations,

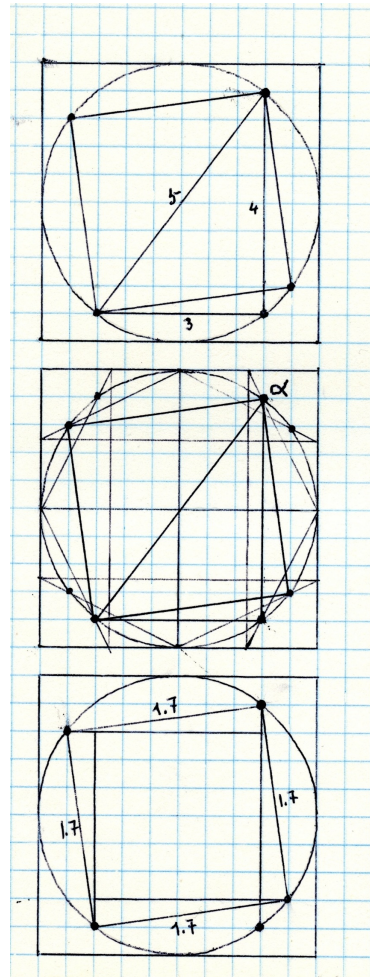


Figure 14: Study for the construction in Figure 12, from one of Almada's notebooks

while some others were exact. As we have said, Almada never produced any mathematical explanation or analysis, which suggests that all his geometrical work was done by extensive, almost obsessive, trial and error. His aim was to exhibit canonical elements and connections between them, without dwelling on mathematical accuracy. Even though he did acknowledge the influence of others (such as Hambidge), he did claim authorship of the constructions he presented.

Beyond the interest of their geometric program, these works are of unquestionable artistic value. They form an unusual (maybe even unique) ensemble of art works with almost exclusively mathematical content, in perfect harmony with an aesthetical intention. For over forty years, Almada experimented on geometry, as a self-taught process, constantly producing works of art, which constitute, on its own, the final format of his theoretical statements.

6 Acknowledgements

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